Imaging cardiac activity by the D-bar method for electrical impedance tomography

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Abstract

A practical D-bar algorithm for reconstructing conductivity changes from EIT data taken on electrodes in a 2D geometry is described. The algorithm is based on the global uniqueness proof of Nachman (1996 Ann. Math. 143 71–96) for the 2D inverse conductivity problem. Results are shown for reconstructions from data collected on electrodes placed around the circumference of a human chest to reconstruct a 2D cross-section of the torso. The images show changes in conductivity during a cardiac cycle.

Keywords

electrical impedance tomography; reconstruction algorithm; cardiac activity; D-bar method

1. Introduction

Electrical impedance tomography (EIT) may provide an inexpensive and portable method for bedside imaging of cardiac activity and pulmonary perfusion. One possible electrode configuration is to place electrodes around the circumference of the subject’s chest and form a 2D cross-sectional image of the conductivity distribution in the plane of the electrodes. In this work, the D-bar method (Siltanen et al 2000) of reconstructing the conductivity is tailored to the application of forming difference images of the conductivity from a reference frame of human chest data. A description and discussion of the method follows in section 2. The images show changes in conductivity during a cardiac cycle. These are the first human-subject images by the D-bar method.


The paper is organized as follows. In section 2 we describe the D-bar algorithm and its implementation for the chest data sets. The experimental procedure is described in section 3.
2. A practical $\partial$ reconstruction method

The algorithm in this paper is based on the global uniqueness proof of Nachman (1996) for the 2D inverse conductivity problem (Calderón 1980). That is, the problem of determining a sufficiently smooth conductivity function $\gamma(x)$ in a 2D domain $\Delta$ from knowledge of the Dirichlet-to-Neumann, or voltage- to-current density map, $\Lambda$. The electric field $u(x)$ satisfies the generalized Laplace equation

$$\nabla \cdot (\gamma(x) \nabla u(x)) = 0, \quad x \in \Omega. \tag{1}$$

The Dirichlet-to-Neumann map is defined by

$$\Lambda : u|_{\partial \Omega} \to \gamma(x) \frac{\partial u}{\partial n}|_{\partial \Omega}. \tag{2}$$

The proof in Nachman (1996) is constructive; that is, it provides equations for the conductivity distribution and intermediate functions in terms of the Dirichlet-to-Neumann data. A numerical realization of the equations in Nachman (1996) was given in Siltanen et al. 2000. This method was further developed and applied to experimental tank data in the subsequent works (Mueller and Siltanen 2003, Isaacson et al. 2004). The reader is referred to Nachman (1996), Siltanen et al. 2000, Mueller and Siltanen (2003), Isaacson et al. 2004 for background and a detailed description of the algorithm and its implementation. A global uniqueness proof for $\gamma \in L^\infty$ based on D-bar techniques is given in Astala and Päivärinta. The terminology D-bar algorithm comes from a phonetic way of writing $\partial$, the partial derivative with respect to the conjugate of a complex variable. This differential operator is defined for a complex variable $z = x + iy$ by

$$\partial_z = \frac{1}{2} \left( \frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right).$$

A key step in the algorithm is the solution of a partial differential equation in terms of the $\partial$ operator with respect to a complex parameter $k$. In this paper we assume some familiarity with the algorithm and focus on its application to experimental data; particularly human chest data. The cross-sectional human chest is modeled by a disk of radius $r$, and the Dirichlet-to-Neumann map on the circle of radius $r$ containing conductivity distribution $\gamma(x)$ is denoted by $\Lambda_{r}$. Note that $\Lambda_{r,1} = r \Lambda_{r,r}$, allowing us to scale the problem to the unit disk. Also, the theoretical work of Nachman assumes $\gamma = 1$ in a neighborhood of the boundary of $\Omega$. In practice, this assumption is necessarily violated by the presence of the electrodes on the skin. Although the algorithm outlined in Nachman’s proof contains an initial step in which the conductivity is reconstructed on the boundary and smoothly extended to 1 on the boundary of a slightly larger domain $\Omega_2$ for which a new Dirichlet-to-Neumann map is then calculated, we omit that step in this work, as in Isaacson et al. 2004. Instead, we compute $\gamma_{\text{best}}$, the best constant conductivity approximation to $\gamma$ from the measured data, and scale $\gamma$ by $\gamma_{\text{best}}$ so that

$$\tilde{\gamma} = \gamma / \gamma_{\text{best}}. \tag{3}$$

Since $\Lambda_\gamma = c \Lambda_{\tilde{\gamma}}$ for $c$ constant, $\Lambda_{\tilde{\gamma}} = \frac{1}{\gamma_{\text{best}}} \Lambda_\gamma$ Thus, we use the map $\Lambda_{\tilde{\gamma}}$ as our data corresponding to the scaled conductivity $\tilde{\gamma}$. After reconstructing $\tilde{\gamma}$, we obtain $\gamma$ from the...
formula $\gamma = \gamma_{\text{best}}$. In the following sections, we will simply use the notation $\gamma$ to represent the scaled conductivity from (3) to avoid cumbersome notation.

The model chest is first discretized into 496 mesh elements, and the conductivity is assumed to be constant in each of these mesh elements. The number of mesh elements is chosen on the basis of the fact that the maximum degrees of freedom for reconstructing the conductivity from $L - 1$ current patterns on $L$ electrodes is $L(L-1)/2$. Here, the ‘Joshua-tree’ mesh, introduced in Cheney et al 1990, was used. This mesh is described in more detail in Cheney et al 1990 and Isaacson et al 2004. We will define $z_j$ to be the radial and angular center of the $j$th element.

The D-bar algorithm used in this paper differs from the D-bar algorithm described in Isaacson et al 2004 in the construction of the scattering transform. Here, a difference map $t_{\text{diff}}$ is computed from which difference images will be reconstructed. The algorithm used in this paper consists of the following steps, which will be described in more detail below.

1. Select a reference frame from the data sets and write a discrete approximation to the difference of Dirichlet-to-Neumann maps $\delta \Lambda_{\gamma, r} = \Lambda_{\gamma, r} - \Lambda_{1, r}$ using the measured current-to-voltage data.
2. Compute the truncated differencing scattering transform $t_{\text{diff}}$ from the discrete Dirichlet-to-Neumann difference map.
3. Numerically solve the D-bar equation (9) in integral form for $\mu_{\text{Rdir}}(z_j, \cdot)$ for each $z_j$ in the chest mesh.
4. Reconstruct the difference image $\gamma(z_j) - \gamma_{\text{ref}}(z_j)$ at each point $z_j$ approximately from $\gamma_{\text{Rdif}}(x) = \mu_{\text{Rdif}}(x, 0)^2$.

2.1. The discrete Dirichlet-to-Neumann map

The reconstruction algorithm requires knowledge of the Dirichlet-to-Neumann map, while in practice, we measure the action of the current-to-voltage map for a finite number of current patterns. Since the applied current patterns constitute a basis for the set of all current patterns that can be applied on $L$ electrodes, a discrete voltage-to-current density map can be approximated from the measurements we have made. Let the electrodes have uniform area $A$. Let $V^k_l$ denote the voltage measured on the $l$th electrode corresponding to the $k$th current pattern $T^k$ and normalized so that $\sum_{l=1}^L V^k_l = 0$, $k = 1, \ldots, L - 1$. Let $T^k$ denote the vector of normalized currents $T^k = \frac{T^k}{\| T^k \|_2}$, where $\| T^k \|_2 = \sqrt{\sum_{l=1}^L (V^k_l)^2}$. The voltages $v^k$ that would result from the normalized currents are then given by $v^k = \frac{\nu^k}{\| T^k \|_2}$. Let $\langle \mathcal{U}(\cdot), \mathcal{W}(\cdot) \rangle = \sum_{j=1}^n \mathcal{U}(\theta_j) \mathcal{W}(\theta_j)$ denote the discrete inner product defined by

$$\langle \mathcal{U}(\cdot), \mathcal{W}(\cdot) \rangle = \sum_{j=1}^n \mathcal{U}(\theta_j) \mathcal{W}(\theta_j).$$

Then the entries of the discrete Neumann-to-Dirichlet map $R_{\gamma, r}$, which is an $L - 1$ by $L - 1$ matrix, are approximated by $R_{\gamma, r}(m, n) = \left[ \frac{v^m}{A} \right]_L$, and the discrete Dirichlet-to-Neumann map $L_{\gamma, r}$, which is an $L - 1$ by $L - 1$ matrix, is approximated by $L_{\gamma, r} = R_{\gamma, r}^{-1}$. The reader is referred to Isaacson et al 2004 for a more detailed derivation of this approximation. We define the map $\delta L_{\gamma, r}$ to be $L_{\gamma, r} - L_{\gamma, \text{ref}, r}$ where $\gamma_{\text{ref}}$ indicates that we are using data from the reference frame.
2.2. The differencing scattering transform

Next we compute the differencing scattering transform $t^{\text{dif}}$, which is a modification of the approximate scattering transform in Isaacson et al 2004 tailored to forming difference images. In Isaacson et al 2004 a series representation of the truncated approximate scattering transform in terms of $L_{\Omega r}$ is derived. This is based on the boundary integral expression introduced in Siltanen et al 2000

$$t^{\text{exp}}(k; \gamma) = \int_{\partial \Omega} e^{ikz}(\Lambda_\gamma - \Lambda^1_1) e^{ikz} d\sigma(z).$$ (4)

Here we use the complex notation $e^{ikz} = e^{i(k_1 + i k_2)(x + iy)}$. Since evaluation of the right-hand side of (4) is stable only for $|k| \leq R$ where the radius $R$ depends on the noise level, $t^{\text{exp}}$ must be truncated. Set

$$t^R_{\text{exp}}(k; \gamma) = \begin{cases} t^{\text{exp}}(k; \gamma) & \text{for } |k| \leq R, \\ 0 & \text{for } |k| > R. \end{cases}$$ (5)

Then

$$t^R_{\text{exp}}(k; \gamma_1, \gamma_2) = t^R_{\text{exp}}(k; \gamma_1) - t^R_{\text{exp}}(k; \gamma_2)$$

$$= \int_{\partial \Omega} e^{ikz}(\Lambda_{\gamma_1} - \Lambda^1_1) e^{ikz} d\sigma(z) - \int_{\partial \Omega} e^{ikz}(\Lambda_{\gamma_2} - \Lambda^1_1) e^{ikz} d\sigma(z)$$

$$= \int_{\partial \Omega} e^{ikz}(\Lambda_{\gamma_1} - \Lambda_{\gamma_2}) e^{ikz} d\sigma(z).$$

The series formulation for $t^{\text{exp}}_R$ derived in Isaacson et al 2004 is used in the actual implementation. The theoretical formula is

$$t^R_{\text{exp}}(k; \gamma) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \langle a_n^{(k)} \rangle_{\gamma} a_n^{(k)} e^{im\theta}, (\Lambda_\gamma - \Lambda^1_1) e^{in\theta}. \rangle.$$

where

$$a_n^{(k)}(k) = \begin{cases} \frac{(i k)^n}{n!}, & n \geq 0, \\ 0, & n < 0, \end{cases}$$

and $\langle \cdot, \cdot \rangle$ denotes the integral inner product. In terms of $\delta L_{\gamma R}$ for $t^{\text{dif}}_R$ this becomes
where \( \Delta \theta = \frac{2\pi}{L} \).

2.3. Solution of the D-bar equation

The approximate scattering transform can be used for approximate reconstruction of the conductivity as explained in Mueller and Siltanen (2003) (see Nachman (1996) for the theory behind the algorithm). An essential step of the algorithm is the solution of a \( \partial \), or D-bar, equation relating the scattering transform to an intermediate function, often called a Faddeev exponentially growing solution after the work by Faddeev (1966) applying such solutions of the Schrödinger equation to problems in quantum mechanics. Nachman (1996) shows that the \( \partial \) equation for the EIT problem can be written in integral form as follows:

\[
\mu(z, s) = 1 + \int \frac{1}{(2\pi)^2} \frac{t(k)}{s-k} e^{-z(k) \mu(z, k)} e^{i \delta L \frac{m}{2} + n \frac{L}{2} + m, n} \, dk_1 \, dk_2.
\]

Replacing the exact scattering transform \( t(k) \) by \( t_{\text{dif}}^{R} \) results in the equation

\[
\mu_{R}^{\text{dif}}(z, s) = 1 + \int \frac{1}{(2\pi)^2} \frac{t_{\text{dif}}^{R}(k)}{s-k} e^{-z(k) \mu_{R}^{\text{dif}}(z, k)} e^{i \delta L \frac{m}{2} + n \frac{L}{2} + m, n} \, dk_1 \, dk_2.
\]

A computational method for the solution of (9) is described in Knudsen et al. (2004). Finally, we find an approximation \( \gamma_{\text{dif}}^{R} \) to \( \gamma - \gamma_{\text{ref}} \) by the formula

\[
\gamma_{\text{dif}}^{R}(x) = \mu_{R}^{\text{dif}}(x, 0)^2.
\]

3. Experimental procedure

The images in this paper were formed from archival data measured by the ACT3 system (Edic et al. 1995) at Rensselaer Polytechnic Institute. ACT 3 is a 32-electrode system operating at 28.8 kHz that applies currents and measures the real and quadrature components of the voltage on all 32 electrodes simultaneously. The reconstructions were plotted using the ACT3 display system which includes a low-pass filter.

In this experiment trigonometric current patterns with an amplitude of 0.85 mA on the 32 electrodes were applied and the resulting voltage was measured. Let \( T_k \) denote the \( k \)th current pattern applied where
\[
\gamma_j^k = \begin{cases} 
M \cos (k \theta), & k = 1, \ldots, \frac{L}{2} - 1 \\
M \cos (m \theta), & k = L/2 \\
M \sin ((k - L/2) \theta), & k = \frac{L}{2} + 1, \ldots, L - 1 
\end{cases}
\]

(10)

and \(M\) is the current amplitude.

The voltage-to-current map \(L_{\gamma, R}\) was constructed by the method described in section 2. Thirty-two electrodes 29 mm high by 24 mm wide were placed around the circumference of the male subject's chest. The chest had a circumference of 90 cm, and so its cross-section was modeled by a circle of radius 14.3 cm. 100 frames of data were collected during breath-holding at 18 frames s\(^{-1}\). A reference frame was chosen corresponding to approximately the midpoint in the cardiac cycle, and the scattering transform \(t_{\text{dif}}^R\) was constructed by formula (7) with \(R = 3.5\). Thus, the reconstructions represent difference images from the reference frame, with red depicting regions more conductive than the reference frame, and blue representing regions less conductive than the reference frame.

4. Results and discussion

In figure 1 a sequence of 24 images, representing approximately one cardiac cycle are displayed. In figure 1, dorsal is at the top, ventral is at the bottom of the images, and the subject's left is on the viewer's right. A movie of the whole reconstructed sequence can be viewed at http://www.math.colostate.edu/~mueller/cardiacsequence.

We selected a sequence of 24 consecutive images, representing about 1.33 s, which was approximately one cardiac cycle at the low heart rate of this subject, 45 beats/min. In the normal resting cardiac cycle, about 0.28 s consists of systole, the contraction phase, and the remainder is diastole, the dilation phase. This image sequence begins in mid-systole. The first three or four images and the last three or four images occur during systole. This cardiac cycle is 23 images long; frames 23 and 24 are nearly identical to frames 1 and 2. This confirms the expectation that systole accounts for about 5/18 = 0.28 s of the cardiac cycle. During systole, there is an abrupt disappearance of a prominent red area at the central part of the front of the chest. This is due to the reduction in size of the highly conductive heart during systole. This area reappears in the fifth frame, and slowly grows in amplitude throughout most of diastole as the heart refills with conductive blood. It again disappears abruptly at about frame 20.

When blood is pumped out of the heart, half leaves the chest promptly and flows to the rest of the body. The other half goes to the lungs, which lie adjacent to the heart. As the relatively conductive blood enters the lungs, their conductivity increases, resulting in the red region near the center of the image starting at frame 23. This region grows rapidly during systole through frame 4. Thereafter, the lung region becomes increasingly blue as it begins slowly to return to its low-conductivity state as blood drains out of the lungs throughout diastole.

There appears to be some unexplained geometric distortion of the images, because we would expect to see two regions representing the lungs, on either side of the heart. These two regions appear as one in these images, although other reconstruction algorithms have resolved two lungs under similar test conditions, as did the D-bar algorithm on the experimental tank data in Isaacson et al 2004. The probable cause for this distortion is modeling the chest by a disc. It is well known that such an approximation can cause artifacts; see Adler et al 1996, Gersing et al 1996, Kolehmainen et al 1997, Kolehmainen et al 2005.
5. Conclusions

The D-bar method presented here provides reconstructions that distinguish between different phases of the cardiac cycle. Such information could be useful for analyzing lung perfusion and diagnosis of pulmonary emboli.

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References


Nachman AI 1993 Global Uniqueness for a Two-Dimensional Inverse Boundary Value Problem (Preprint Series No. 19) (Department of Mathematics, University of Rochester)


Figure 1.
A sequence of 24 difference images from human data representing approximately one cardiac cycle reconstructed by the D-bar method. Dorsal is at the top and the subject’s left is at the viewer’s right.

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